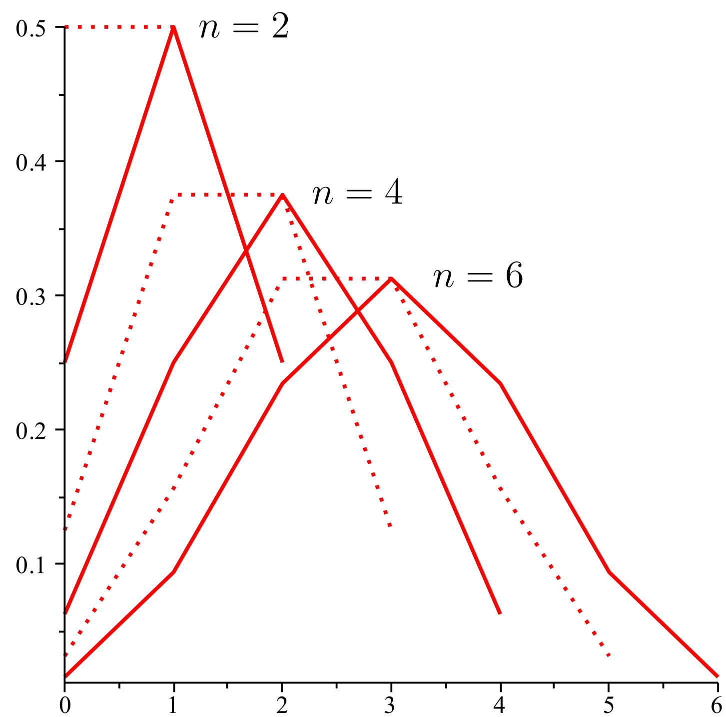
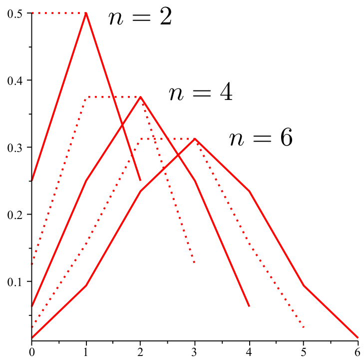
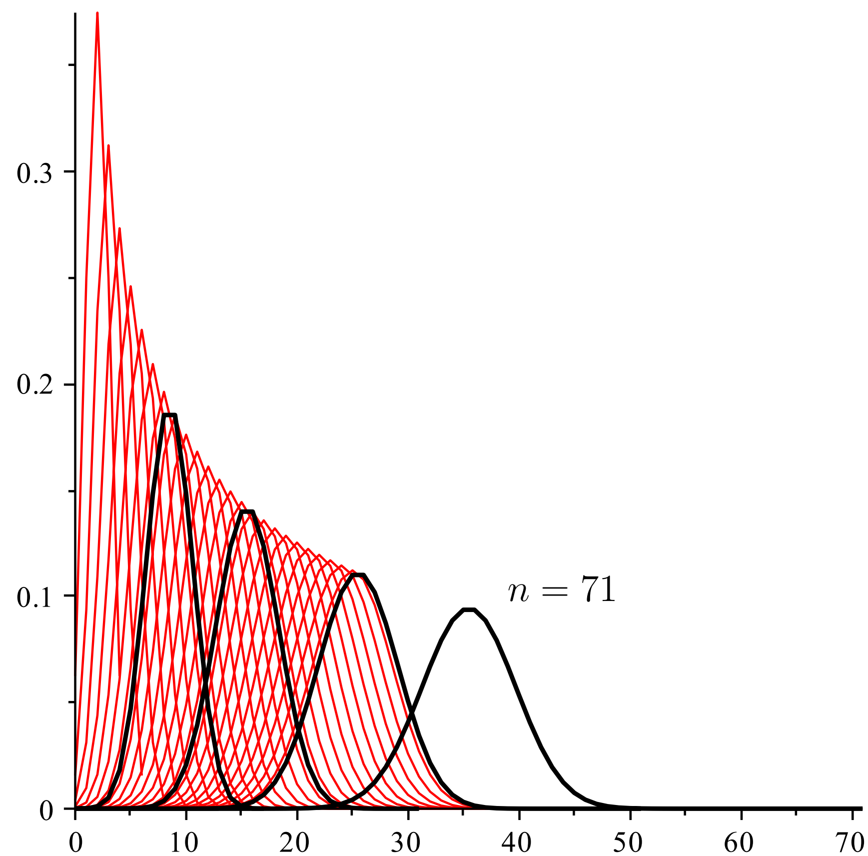
n jets à pile ou face : distribution des gains (1)



$$\left\{ \Big(k, \mathbf{Proba}(S_n = k)\Big) \right\}$$

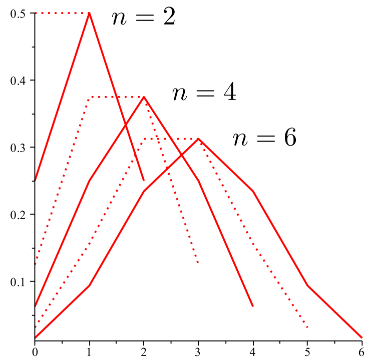
n jets à pile ou face : distribution des gains (2)

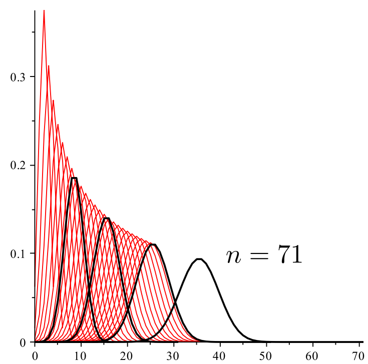




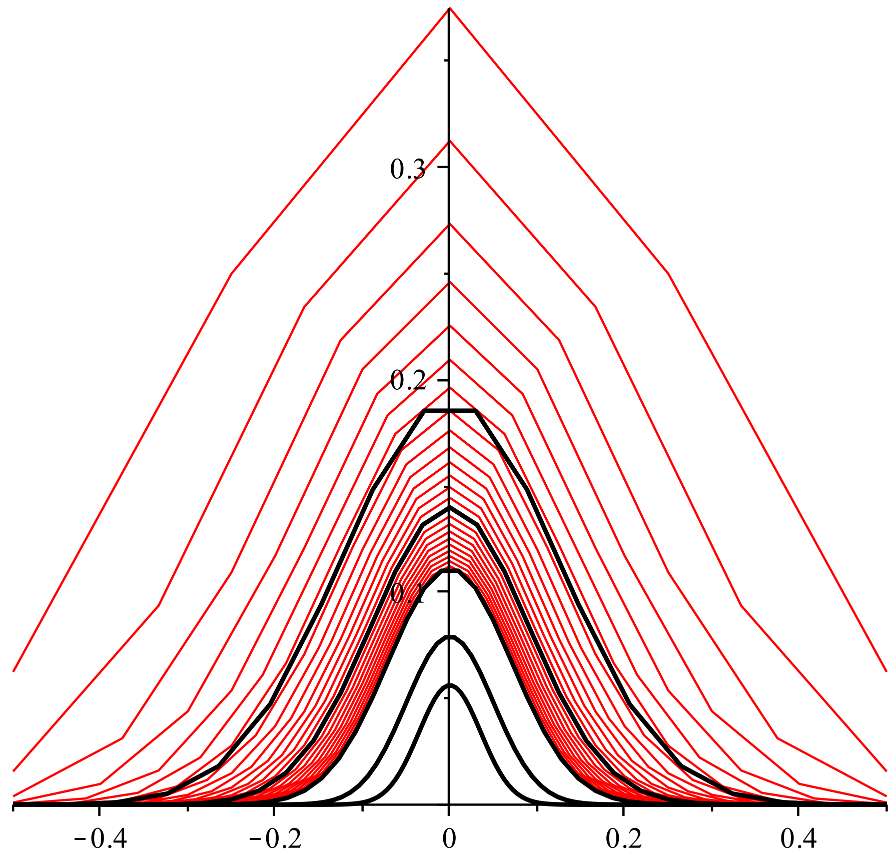
$$\left\{ \left(k, \mathbf{Proba}(S_n = k) \right) \right\}$$

n jets à pile ou face : distribution des gains (3)



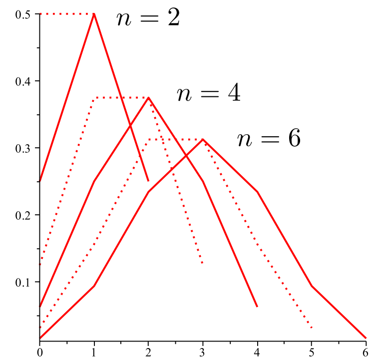


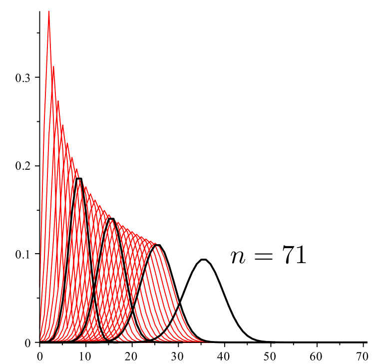
$$\left\{ \left(k, \mathbf{Proba}(S_n = k)\right) \right\}$$

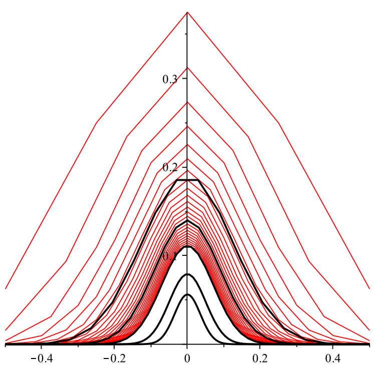


$$\left\{ \left(\frac{k}{n} - \frac{1}{2}, \mathbf{Proba}(S_n = k)\right) \right\}$$

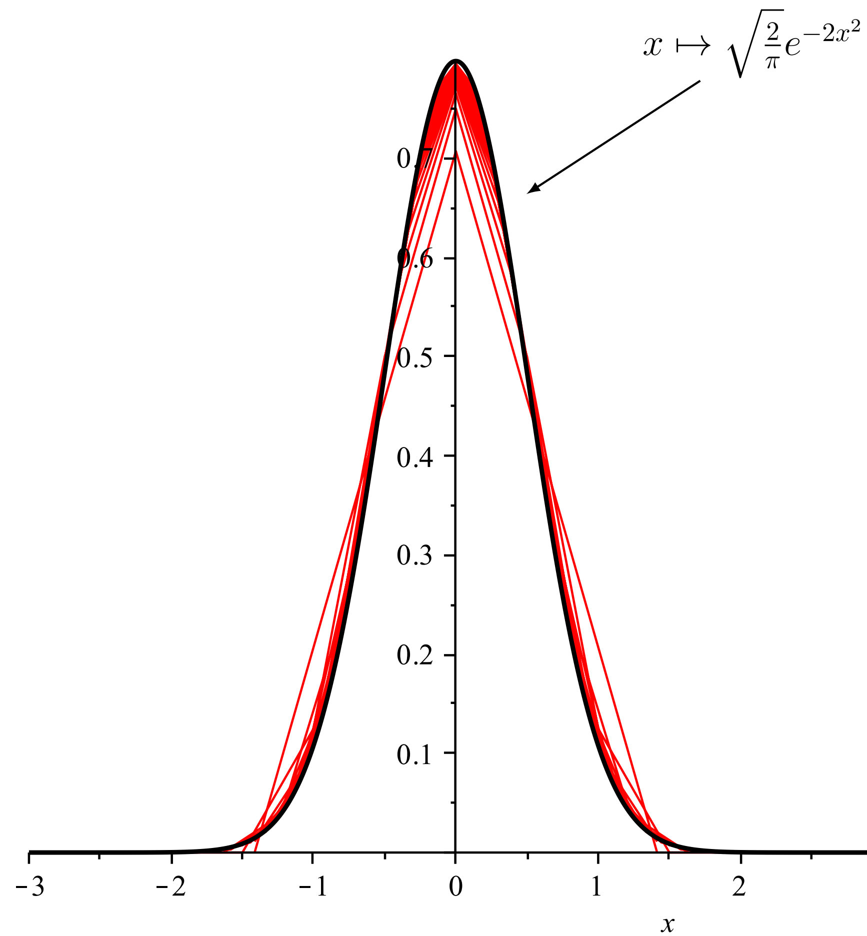
n jets à pile ou face : distribution des gains (4)







$\left\{\left(\frac{k}{n}-\frac{1}{2}, \mathbf{Proba}(S_n=k)\right)\right\}$



$$\left\{\ \left(\frac{k-\frac{n}{2}}{\sqrt{n}}, \sqrt{n}\,\mathbf{Proba}(S_n=k)\right)\ \right\}$$

Somme des carrés des inverses (1)

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \cdots = \ ???$$

Somme des carrés des inverses (2)

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \cdots = \ ?$$

Deux termes : $1 + \frac{1}{2^2} = \frac{5}{4} = {\color{red}1,25}$

Somme des carrés des inverses (3)

$$1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\frac{1}{6^2}+\cdots = \ ?$$

Trois termes : $1+\frac{1}{2^2}+\frac{1}{3^2}=\frac{49}{36}\approx 1,3611111111$

Quatre termes : $1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}=\frac{205}{144}\approx 1,4236111111$

Dix termes : $1+\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{10^2}=\frac{1968329}{1968329}\approx 1.644934068$

Et après ???

Somme des carrés des inverses (4)

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \cdots = \ ?$$

Cent termes : $1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{100^2} \approx 1.634983900$

Mille termes : $1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{1000^2} \approx 1.643934568$

Un million de termes : $1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{(10^6)^2} \approx 1.644933068$

Bon, et après ?

Somme des carrés des inverses (5)

$$\sqrt{6 \times \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \cdots\right)} = \; ???$$

Somme des carrés des inverses (6)

$$\sqrt{6 \times \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \cdots\right)} = \ ?$$

Cent termes : $\sqrt{6 \times \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{100^2}\right)} \approx 3.132076531$

Mille termes : $\sqrt{6 \times \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{1000^2}\right)} \approx 3.140638057$

Un million : $\sqrt{6 \times \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{(10^6)^2}\right)} \approx 3.141591699$

Somme des carrés des inverses (7)

Un milliard de termes :

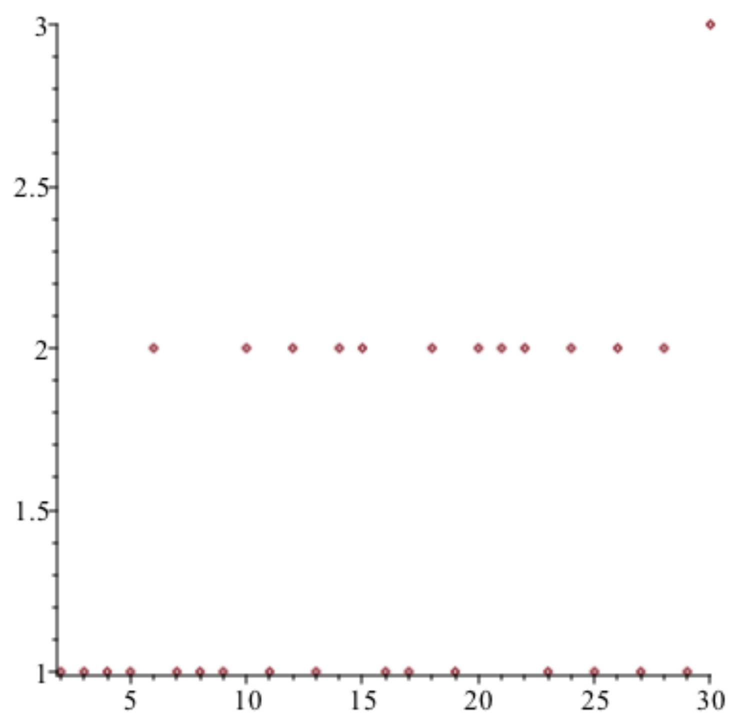
$$\sqrt{6 \times \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{(10^6)^2}\right)} \approx 3.14159265263486$$

A vrai dire, en convergeant très lentement,

$$\boxed{1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \cdots = \frac{\pi^2}{6}}$$

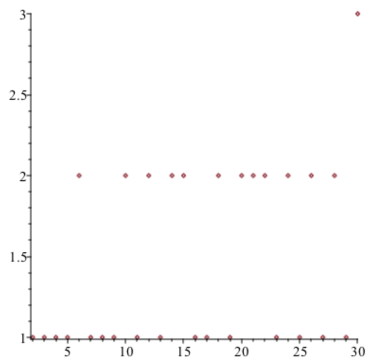
Combien de facteurs premiers ? (1)

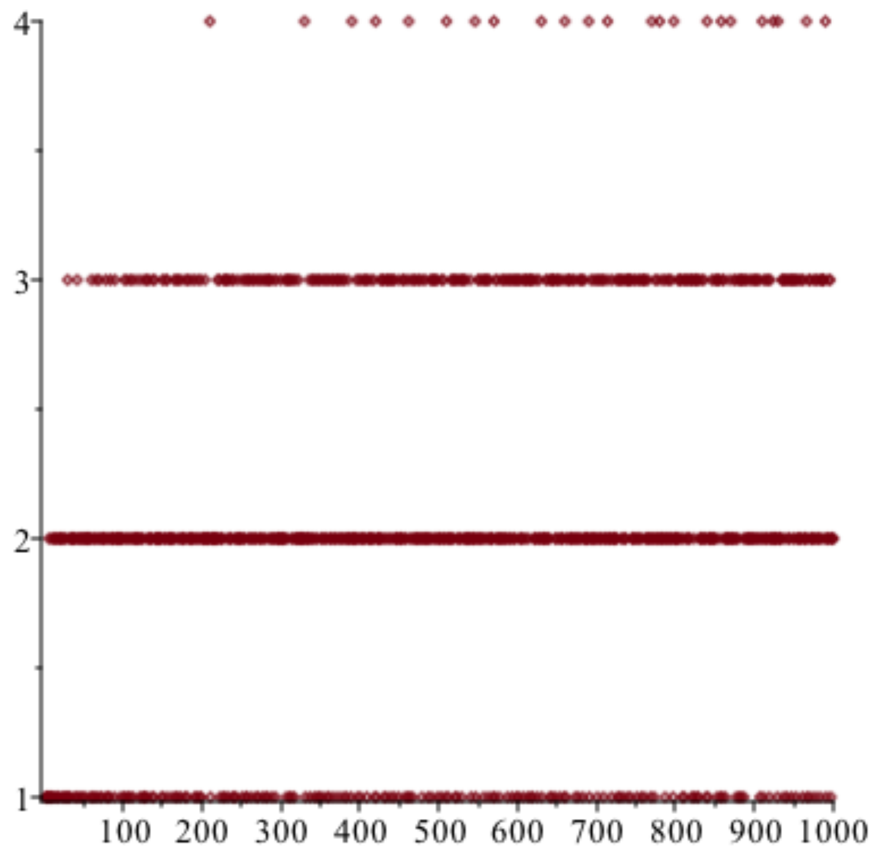
$\omega(n)$ est le nombre de facteurs premiers distincts de n.



$\omega(n), 2 \leq n \leq 30$

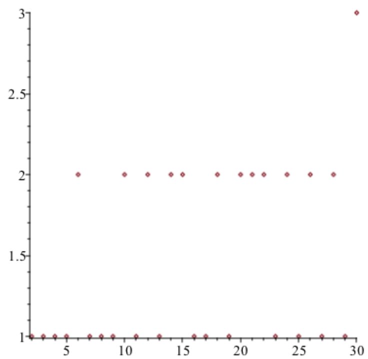
Combien de facteurs premiers ? (2)

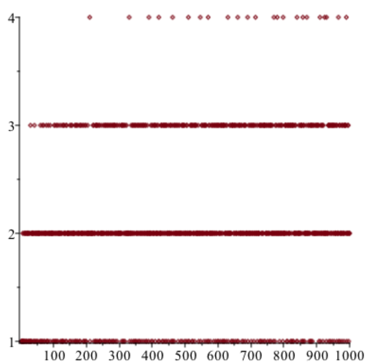


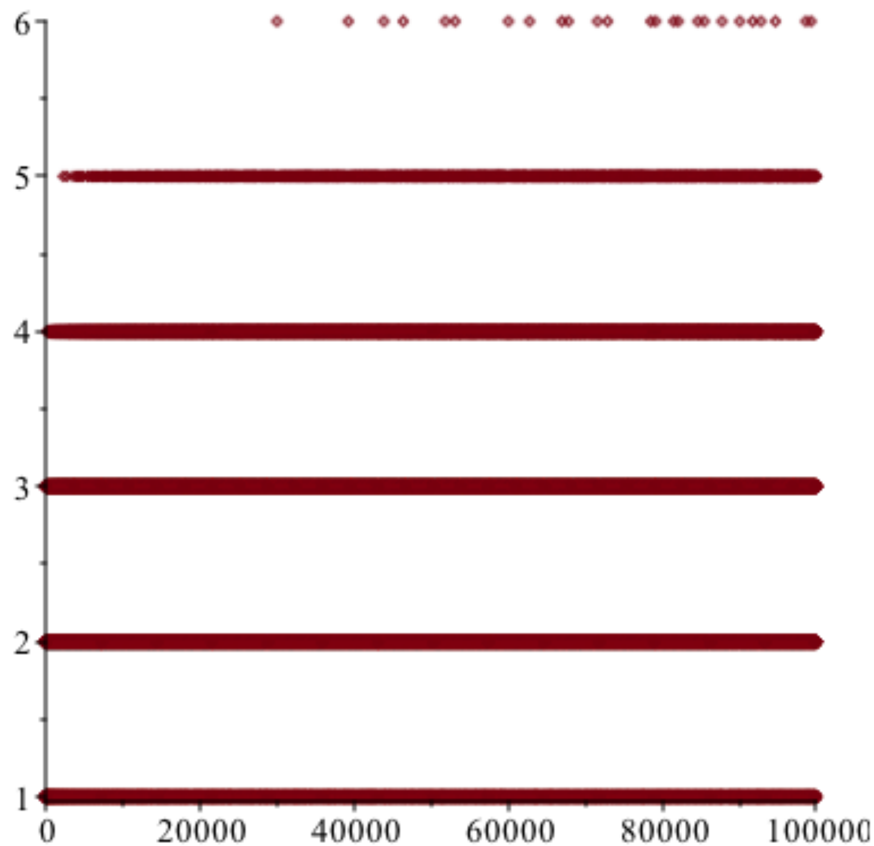


$\omega(n), 2 \leq n \leq 1000$

Combien de facteurs premiers ? (3)



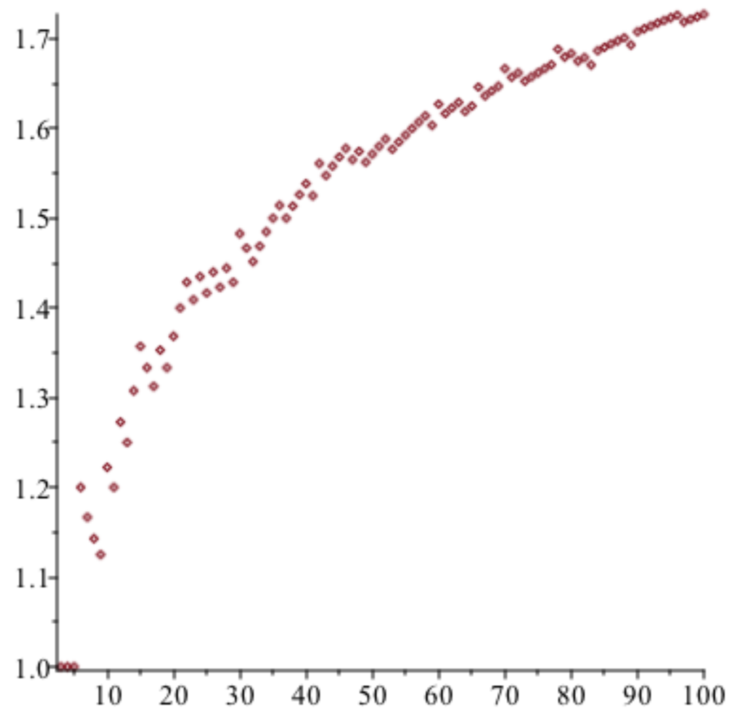




$\omega(n), 2 \leq n \leq 10^5$

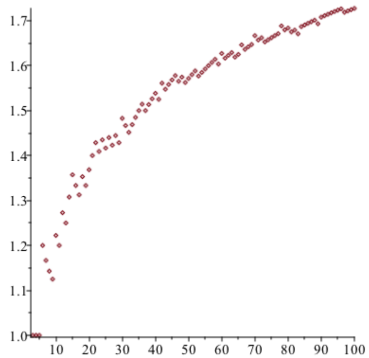
Combien de facteurs premiers ? (4)

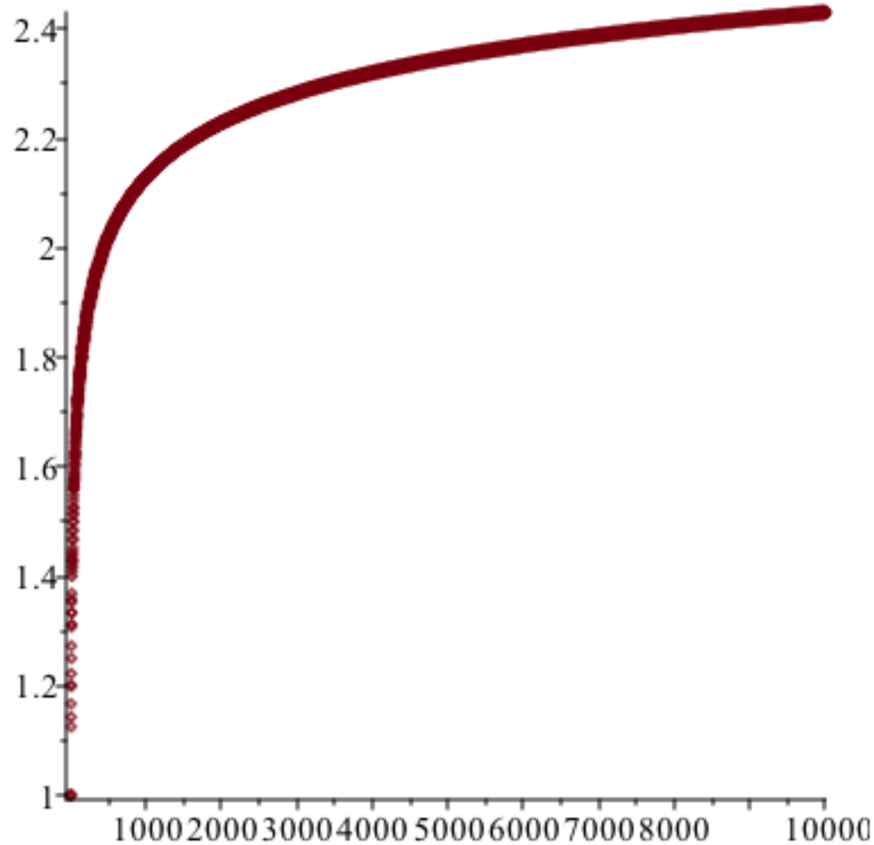
$\overline{\omega}(n)$ est la moyenne des $\omega(k)$, $2 \leq k \leq n$.



$\overline{\omega}(n), 2 \leq n \leq 100$

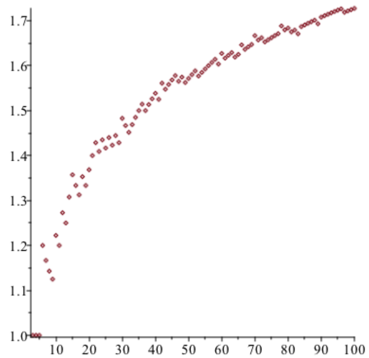
Combien de facteurs premiers ? (5)

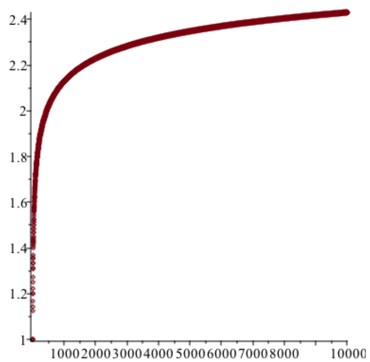


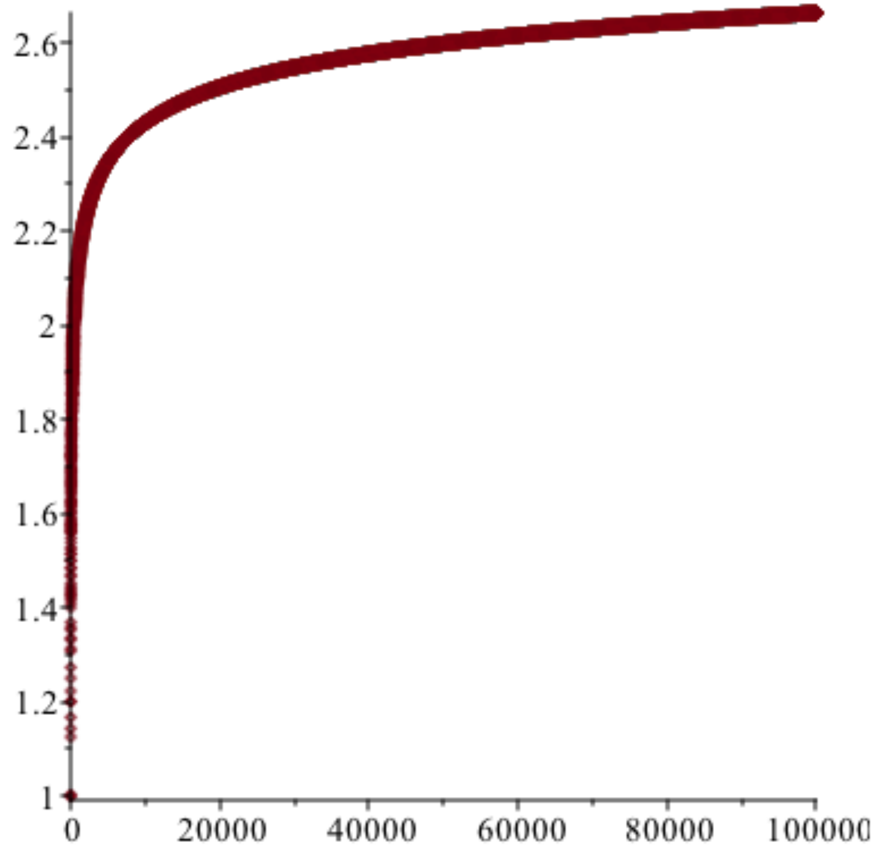


$\overline{\omega}(n), 2 \leq n \leq 1000$

Combien de facteurs premiers ? (6)

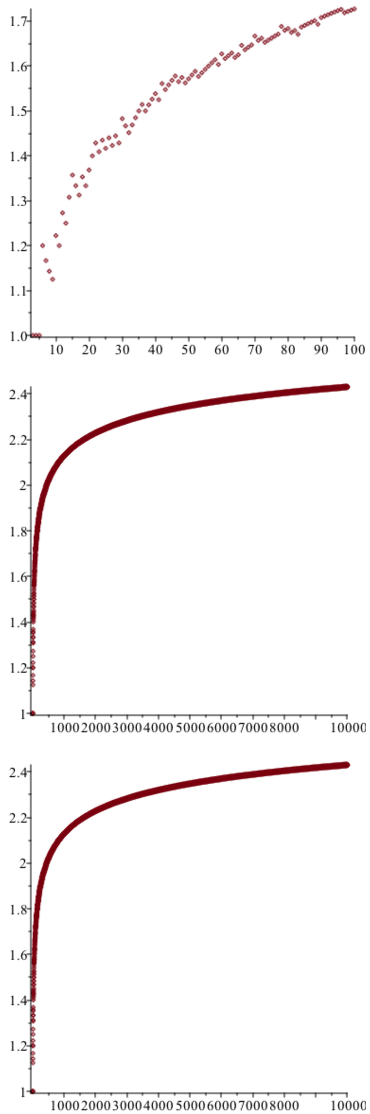




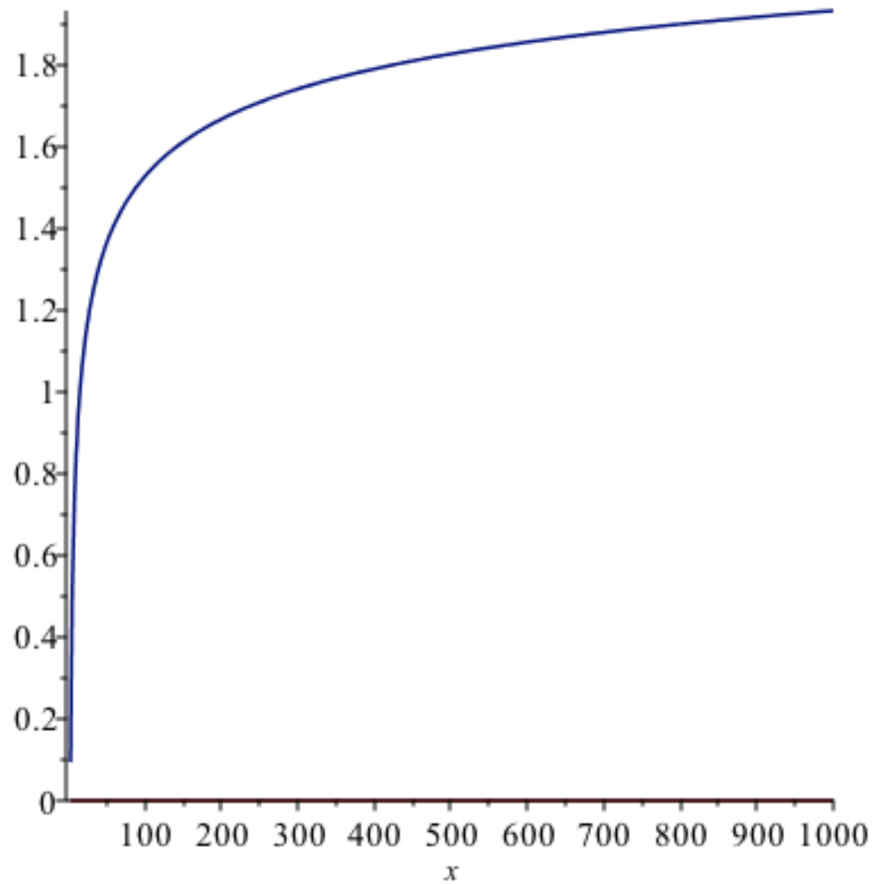


$\overline{\omega}(n), 2 \leq n \leq 10^5$

Combien de facteurs premiers ? (7)

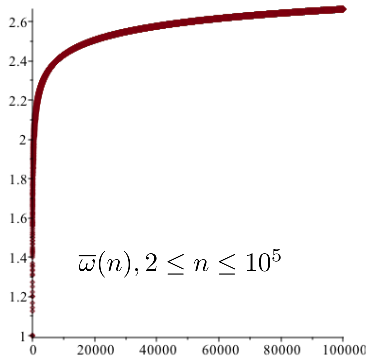


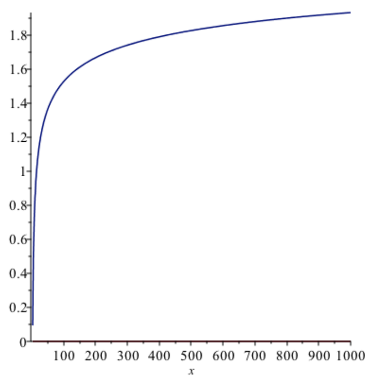
$\overline{\omega}(n), 2 \leq n \leq 10^5$

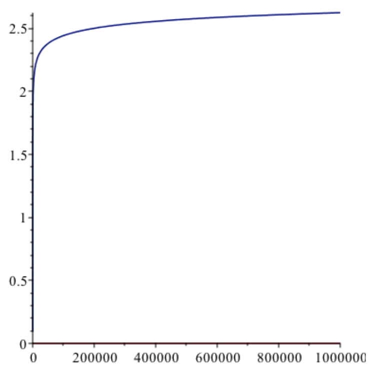


$x \mapsto \ln \ln x, \;\; 2 \leq x \leq 1000$

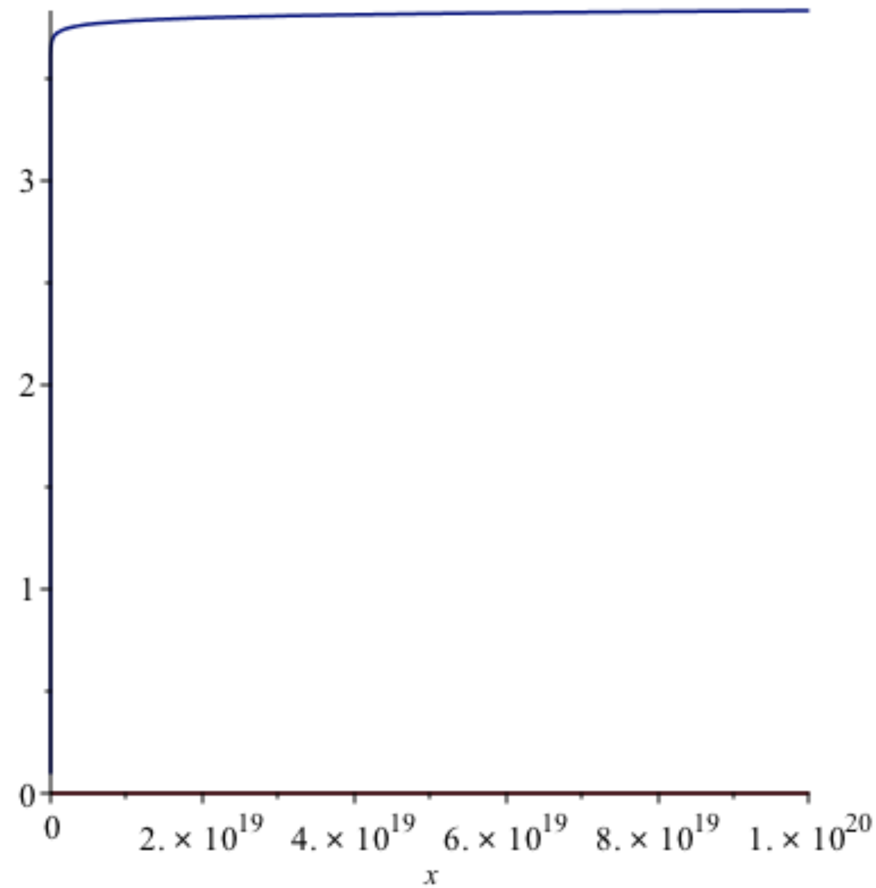
Combien de facteurs premiers ? (8)







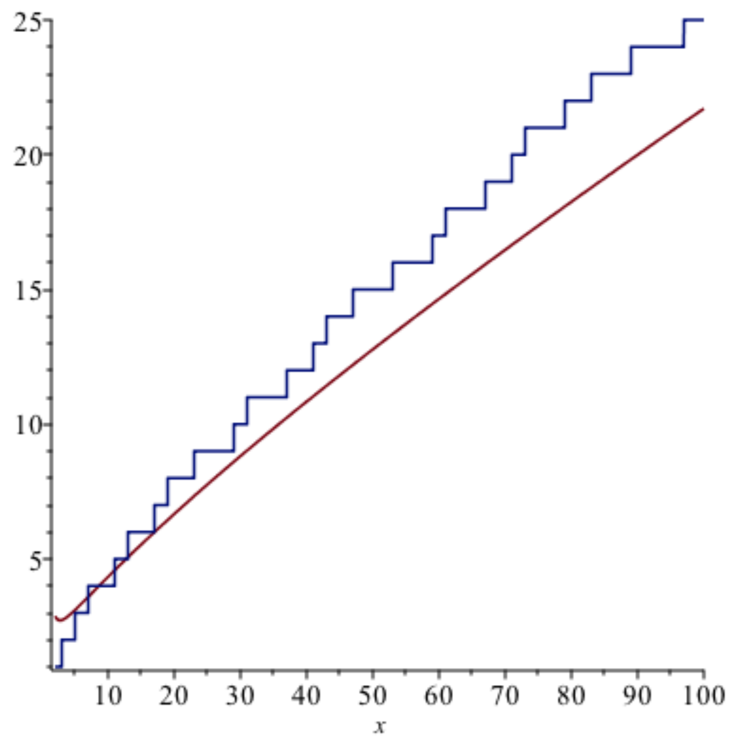
$x \mapsto \ln \ln x$



$x \mapsto \ln \ln x, \;\; 2 \leq x \leq 1000$

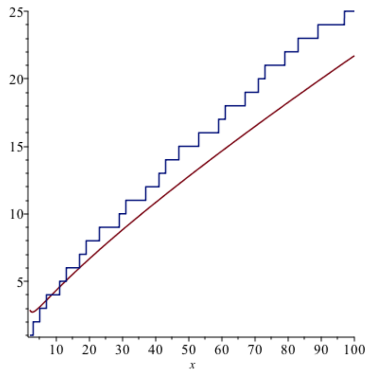
Fonction π (1)

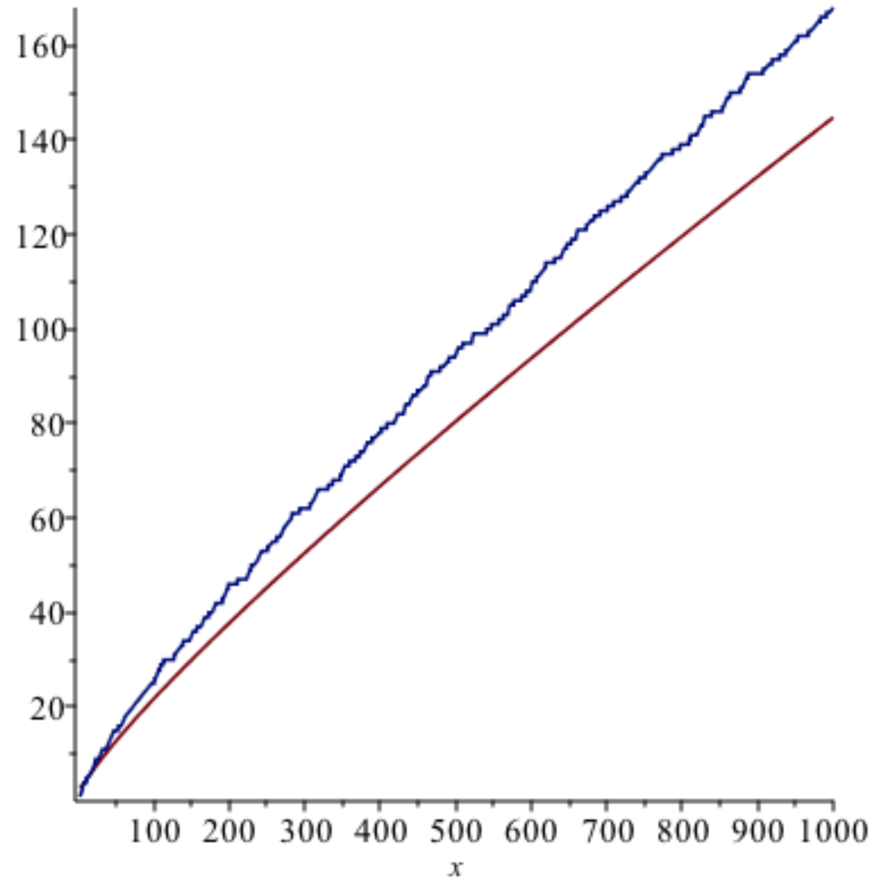
$\pi(x)$ est le nombre de nombres premiers $\leq x$.



$\pi(x)$ et $\frac{x}{\ln x}$, $2 \leq x \leq 100$

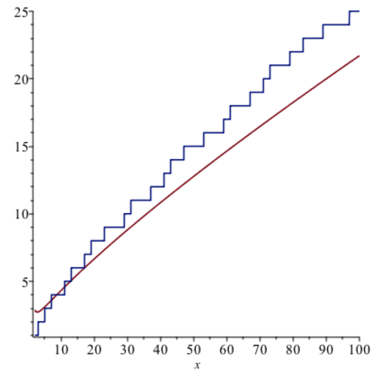
Fonction π (2)

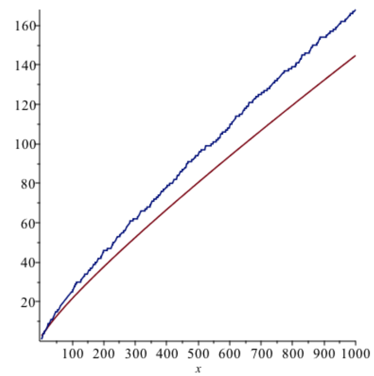


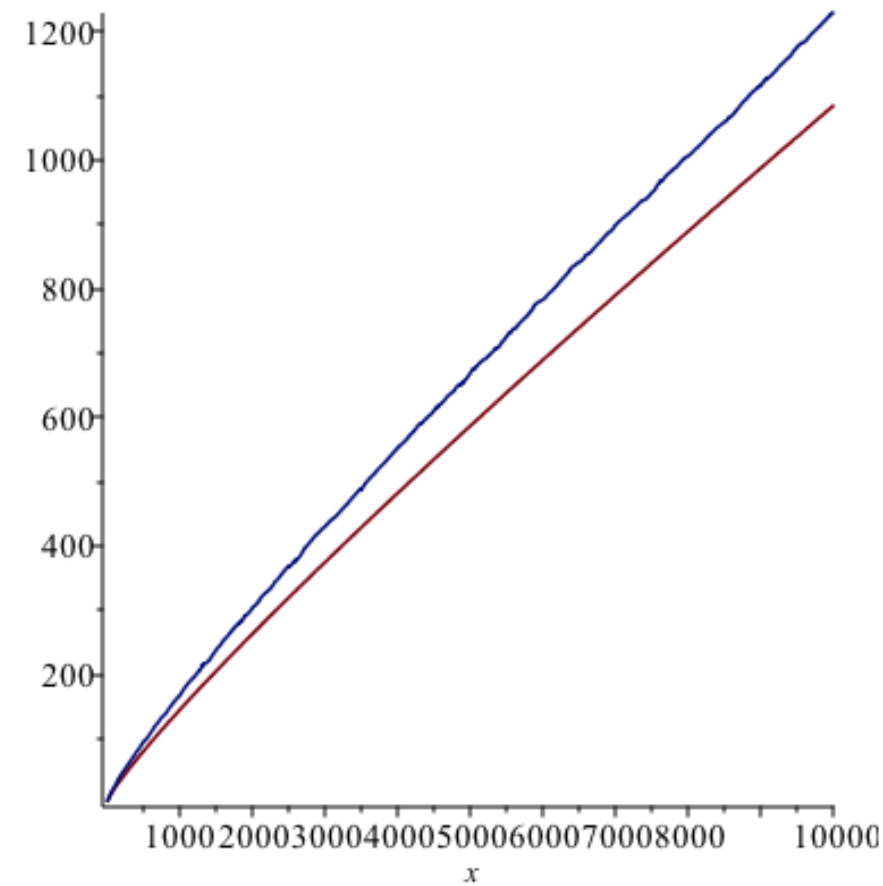


$\pi(x)$ et $\frac{x}{\ln x}$, $2 \leq x \leq 1000$

Fonction π (3)



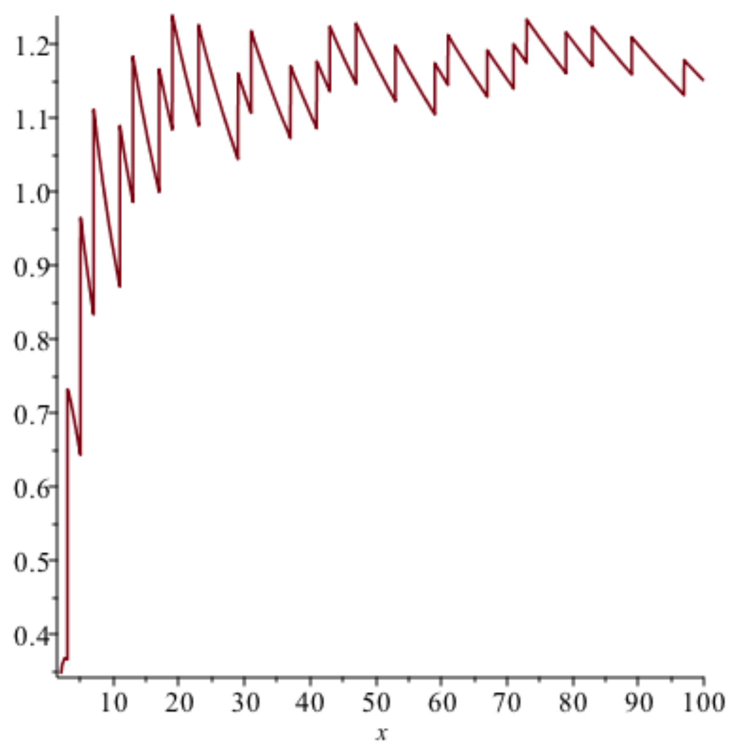




$\pi(x)$ et $\frac{x}{\ln x}$, $2 \leq x \leq 10^5$

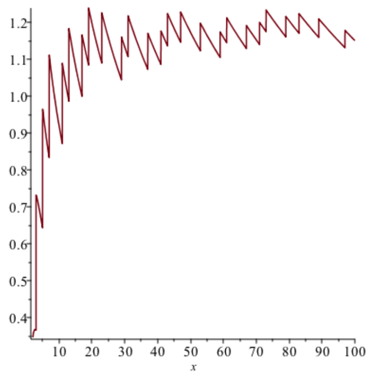
Fonction π (4)

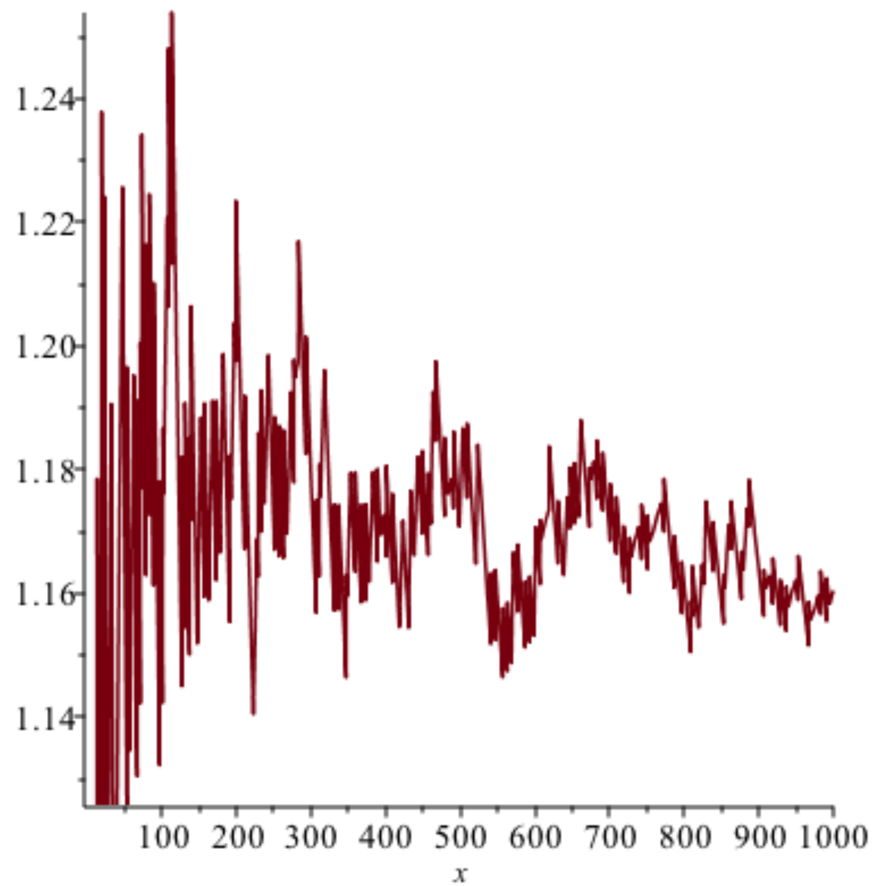
$\pi(x) \sim \frac{x}{\ln x}$ lorsque x tend vers $+\infty$.



$\pi(x)/\frac{x}{\ln x},\ 2 \leq x \leq 100$

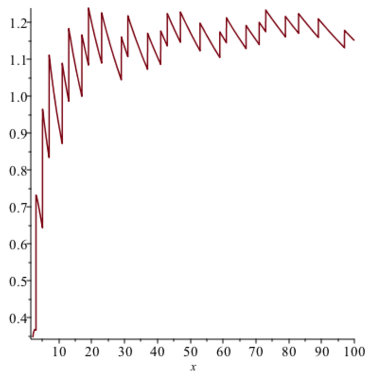
Fonction π (5)

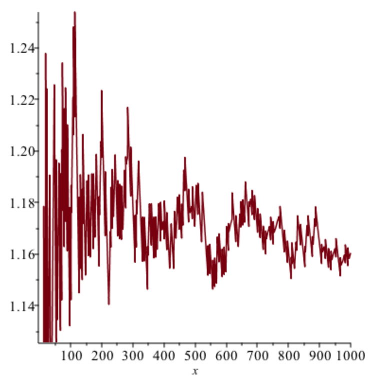




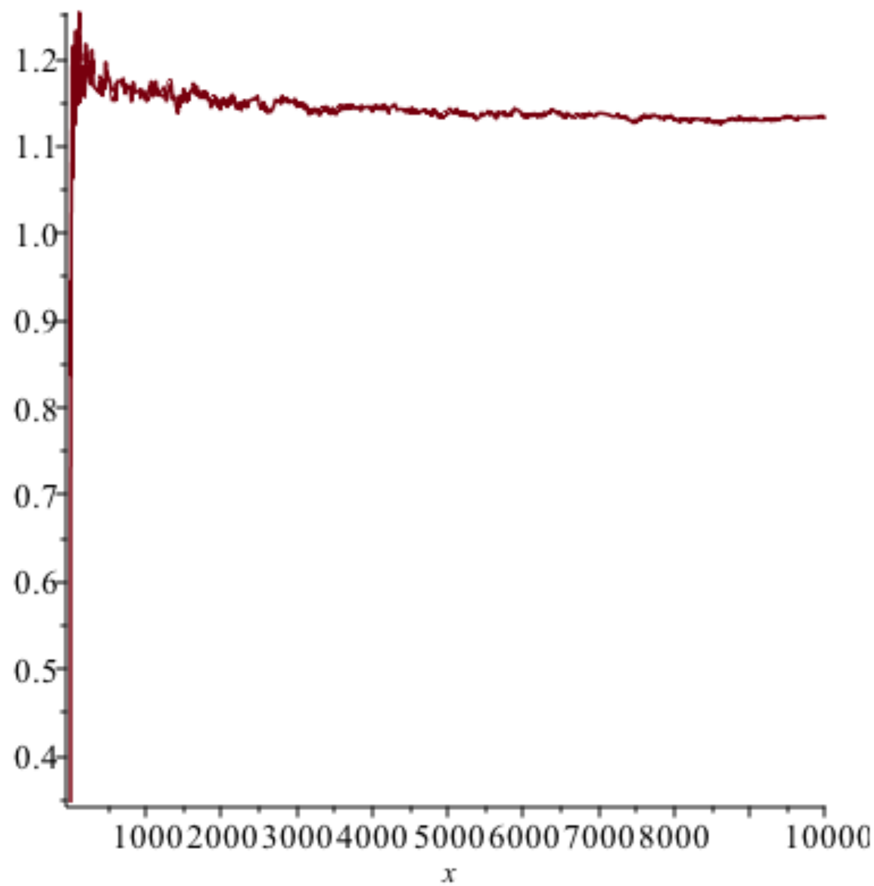
$\pi(x)/\frac{x}{\ln x},\ 2 \leq x \leq 1000$

Fonction π (6)





$\pi(x)/\frac{x}{\ln x} \to 1$
lorsque $x \to +\infty$,
mais... lentement !



$\pi(x)/\frac{x}{\ln x},\ 2 \leq x \leq 10^5$